Corridors and distancing

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Statement of the problem

The problem was initially stated in French as part of a list of 11 problems suggested for the MaSuD (Mathematics for Sustainable Development) 2020-2021 project.

The topic of the problem is the following:

We will model the movement of individuals between two rooms by means of a corridor. At the beginning, we prevent any collision, then we establish social distancing areas around each person. These areas must not intersect. Experiments will show that some corridor shapes or sizes are better suited than others. One possible application would consist in analyzing the maximum social distancing allowed by the passage areas in your high school, without disturbing the flow of students.

Introduction

For this project we constructed a corridor model and chose to analyze the efficiency of such a corridor based only on its SHAPE.

We considered students to be point-like, and denoted by r the minimum recommended distance between students ($r = 1m$ according to the WHO and $r = 6ft \cong 1.8m$ according to the CDC).

With these considerations in mind, we proceed to define the efficiency of a corridor based on shape, maximize this efficiency, and then compute the efficiency of some well-known corridor shapes such as a corner corridor and a circular arc corridor.

Results

We worked under the following assumptions:

Assumption 1: The students move orderly, into lines of students going the same way. In these student lines, the distance between any two consecutive students is the same.

Assumption 2: A corridor must allow students to move both ways, therefore every corridor will support at least two student lines.

Assumption 3: All students have the same velocity v at all times.

We defined the student flux as the number of students passing through the corridor in a time unit:

$$q = \frac{\Delta N}{\Delta t}$$

Using physical world considerations, we found that the efficiency (e) is proportional to the student flux (q) and inverse proportional to the surface area of the corridor (S).

Thus, we defined:

$$e = \frac{q}{S} = \frac{q}{l \cdot w} = \frac{\Delta N}{l \cdot w \cdot \Delta t}$$

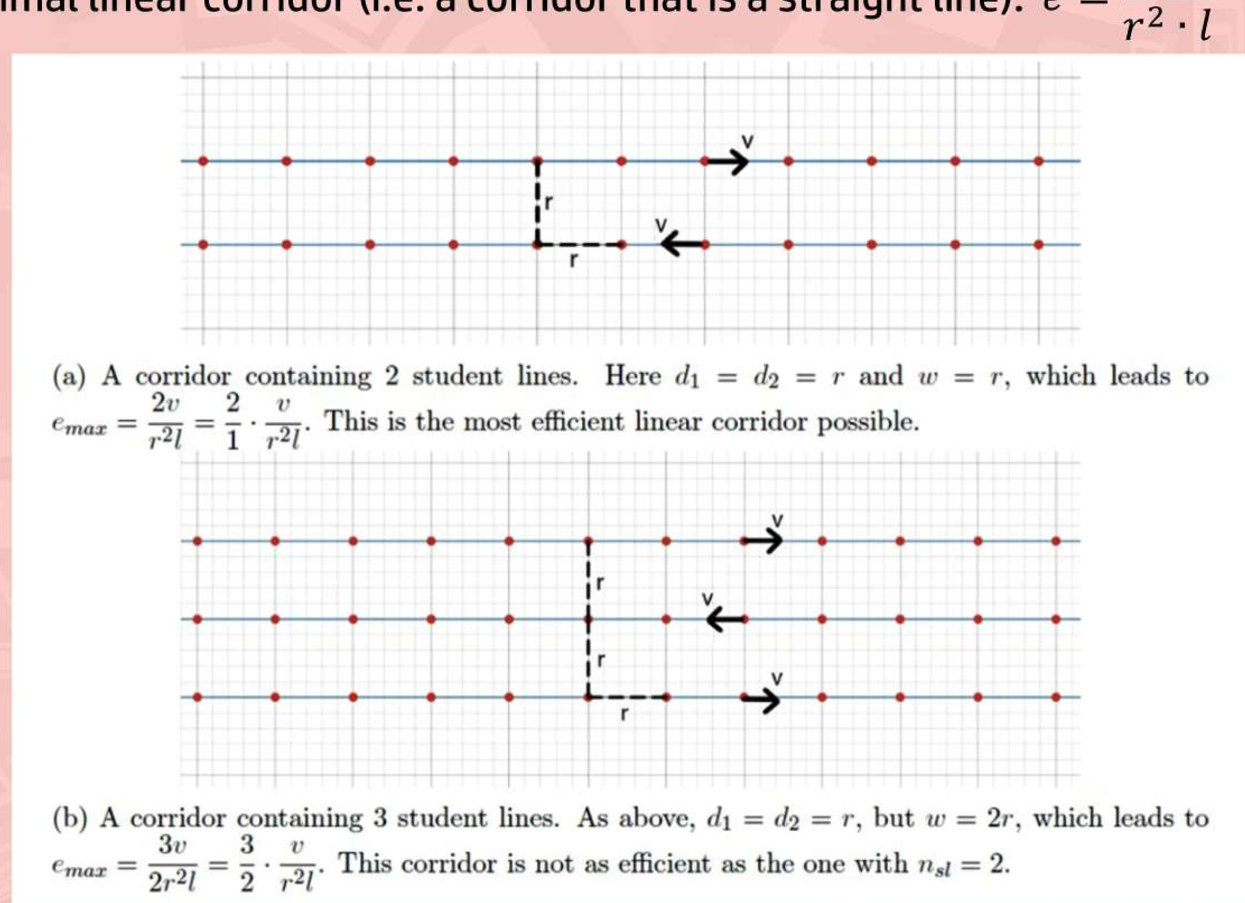
Where l is the length of the corridor and $w \geq r$ is its width (constant for the entire length of the corridor).

Denoting by d_1 the distance between consecutive students in the same student line and using basic inequalities, we obtained an interesting and counterintuitive result:

The most efficient corridor is the one with the smallest WIDTH $w = r$ (this corridor contains exactly 2 student lines, along the walls).

The subsequent efficiencies are:

- for an optimal non-linear corridor (i.e. a corridor that is not a straight line): $e = \frac{2v}{d_1 \cdot r \cdot l}$
- for an optimal linear corridor (i.e. a corridor that is a straight line): $e = \frac{2v}{r^2 \cdot l}$



(a) A corridor containing 2 student lines. Here $d_1 = d_2 = r$ and $w = r$, which leads to $e_{max} = \frac{2v}{r^2 l} = \frac{2}{1} \cdot \frac{v}{r^2 l}$. This is the most efficient linear corridor possible.

(b) A corridor containing 3 student lines. As above, $d_1 = d_2 = r$, but $w = 2r$, which leads to $e_{max} = \frac{3v}{2r^2 l} = \frac{3}{2} \cdot \frac{v}{r^2 l}$. This corridor is not as efficient as the one with $n_{sl} = 2$.

We then proceeded to prove that the efficiency of a non-linear corridor is less than that of its linear counterpart (because the smallest distance between two points is in a straight line).

Therefore, to evaluate the efficiencies of non-linear corridors we introduced the **relative efficiency η.**

For the optimal case with $w = r$, the relative efficiency turns out to be:

$$\eta = \frac{r}{d_1} \cdot \frac{l_L}{l_{NL}}$$

where l_L is the length of the linear corridor and l_{NL} is the length of the non-linear corridor ($l_{NL} > l_L$).

Corner corridor

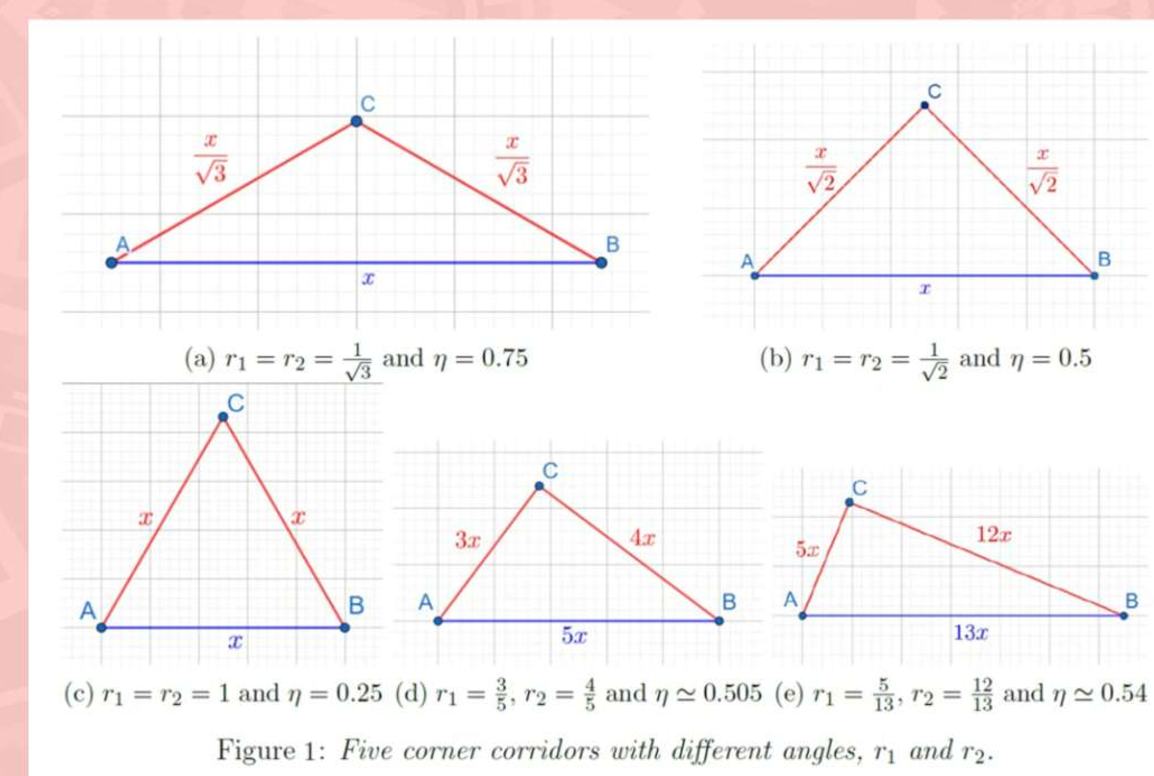
We considered a corner corridor between points A and B to be a path A – C – B with the following ratios: $\frac{AC}{AB} = r_1$ and $\frac{CB}{AB} = r_2$.

Then, for the optimal case ($w = r$):

$$\eta = \frac{1}{2(r_1 + r_2)} \cdot \sqrt{\frac{1 - (r_1 - r_2)^2}{r_1 r_2}}$$

Particular cases:

- the equilateral triangle has $\eta = 0.25$
- the isosceles right triangle has $\eta = 0.5$
- the isosceles triangle with an angle of 120° has $\eta = 0.75$
- the 3-4-5 right triangle has $\eta \cong 0.51$
- the 5-12-13 right triangle has $\eta \cong 0.54$



(a) $r_1 = r_2 = \frac{1}{\sqrt{3}}$ and $\eta = 0.75$ (b) $r_1 = r_2 = \frac{1}{\sqrt{2}}$ and $\eta = 0.5$

(c) $r_1 = r_2 = 1$ and $\eta = 0.25$ (d) $r_1 = \frac{3}{5}$, $r_2 = \frac{4}{5}$ and $\eta \simeq 0.505$ (e) $r_1 = \frac{5}{13}$, $r_2 = \frac{12}{13}$ and $\eta \simeq 0.54$

Figure 1: *Five corner corridors with different angles, r_1 and r_2.*

Circular arc corridor

We considered a circular arc corridor between points A and B to be a circular arc path belonging to a circle of radius R and having a central angle of α (in radians).

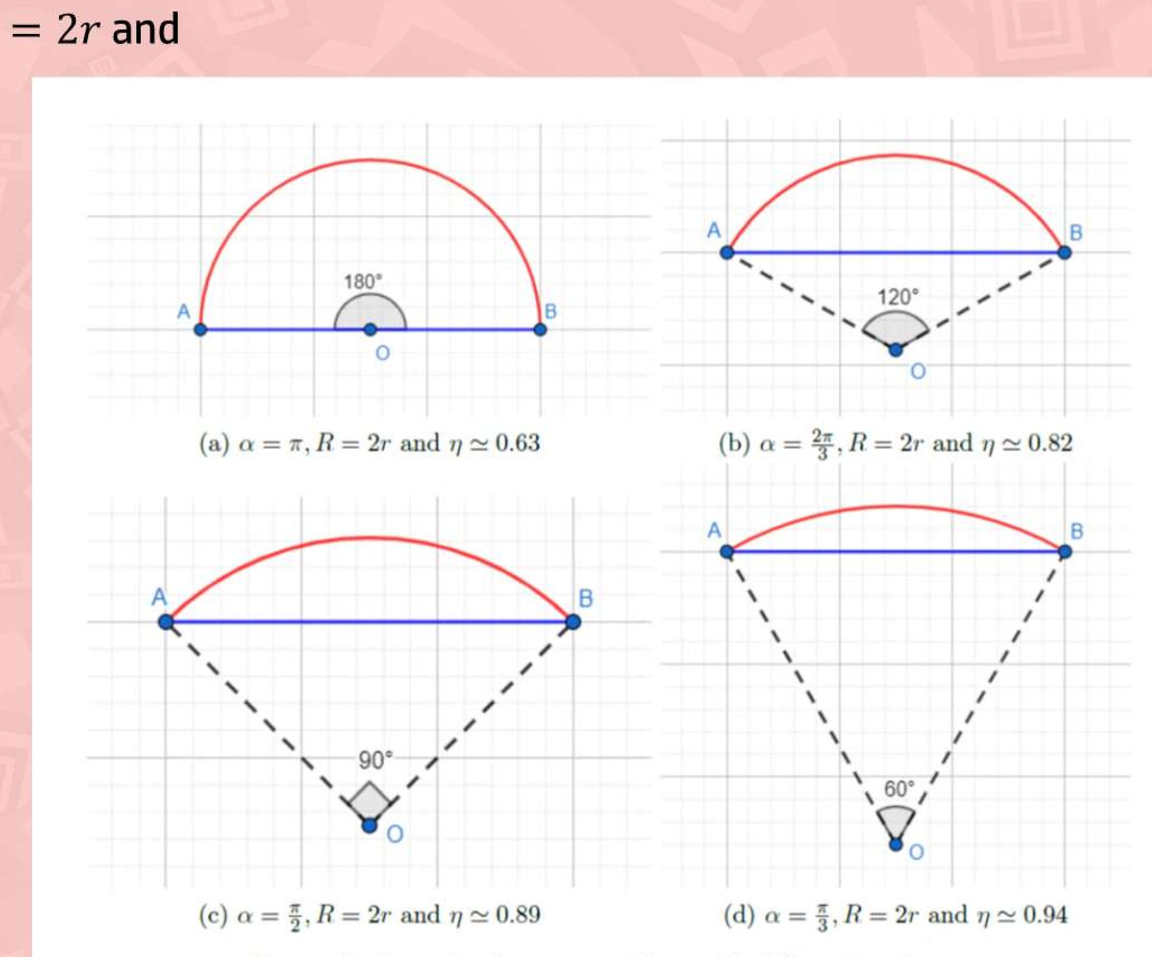
Then, for the optimal case ($w = r$):

$$\eta = \frac{\frac{r}{2R}}{\arcsin \frac{r}{2R}} \cdot \frac{\sin \frac{\alpha}{2}}{\frac{\alpha}{2}}$$

Particular cases:

For the particular case of $R = 2r$ and

- $\alpha = \pi$ we get $\eta \cong 0.63$
- $\alpha = \frac{2\pi}{3}$ we get $\eta \cong 0.82$
- $\alpha = \frac{\pi}{2}$ we get $\eta \cong 0.89$
- $\alpha = \frac{\pi}{3}$ we get $\eta \cong 0.94$



(a) $\alpha = \pi, R = 2r$ and $\eta \simeq 0.63$ (b) $\alpha = \frac{2\pi}{3}, R = 2r$ and $\eta \simeq 0.82$

(c) $\alpha = \frac{\pi}{2}, R = 2r$ and $\eta \simeq 0.89$ (d) $\alpha = \frac{\pi}{3}, R = 2r$ and $\eta \simeq 0.94$

Figure 2: *Four circular arc corridors with different angles α.*

Conclusion and generalization

In this project we have created a model for analyzing the efficiency of a corridor based on its shape, gave general formulas for the efficiency, and particularized these formulas for popular corridor shape. Although our school has only straight line and corner corridors, we computed the efficiency for the circular corridor as well.

One possible generalization would consist in studying the case in which students are not point-like, having a radius $0 < r' \leq r$. Then, the formulas from above would become:

$$e = \frac{2v}{d_1 \cdot (r + 2r') \cdot l} \quad \text{and} \quad e = \frac{2v}{r \cdot (r + 2r') \cdot l}.$$

Other ideas for possible future generalizations are: extending the model for corridors with varying width, studying the case in which two student lines moving in the same direction are closer than r apart, and computing efficiencies for more complex corridor shapes (such as a polygonal contour or an elliptic arc).