Bacteria attacks

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Introduction

In a bacterial culture some of the bacteria organisms are producing a toxic substance that kills them. The change in population is modeled by a differential equation, where the growth rate is proportional to the existing population and decreases at a rate proportional to the concentration of the toxic substance.

$$dN = k \cdot N(t) \cdot (1 - a \cdot T(t))$$
$$dT(t) = r \cdot N(t)$$
$$N(t) = population\ of\ bacteria$$
$$T(t) = concentration\ of\ the\ toxic\ substance$$
$$a = deadliness$$
$$k = natality$$
$$r = rate\ of\ toxin\ production\ per\ organism$$

Methods

Out of the two possible ways to approach the problem (mathematically, by solving the system of two equations, or iteratively through a computer simulation) we decided to use the computer based one. This way, the simulation generates multiple discrete moments in the bacteria lifecycle, through successive iterations. This is a good approximation to what a real situation would look like, as it takes small enough steps in the process, especially if the simulation is run over a longer period.

Results

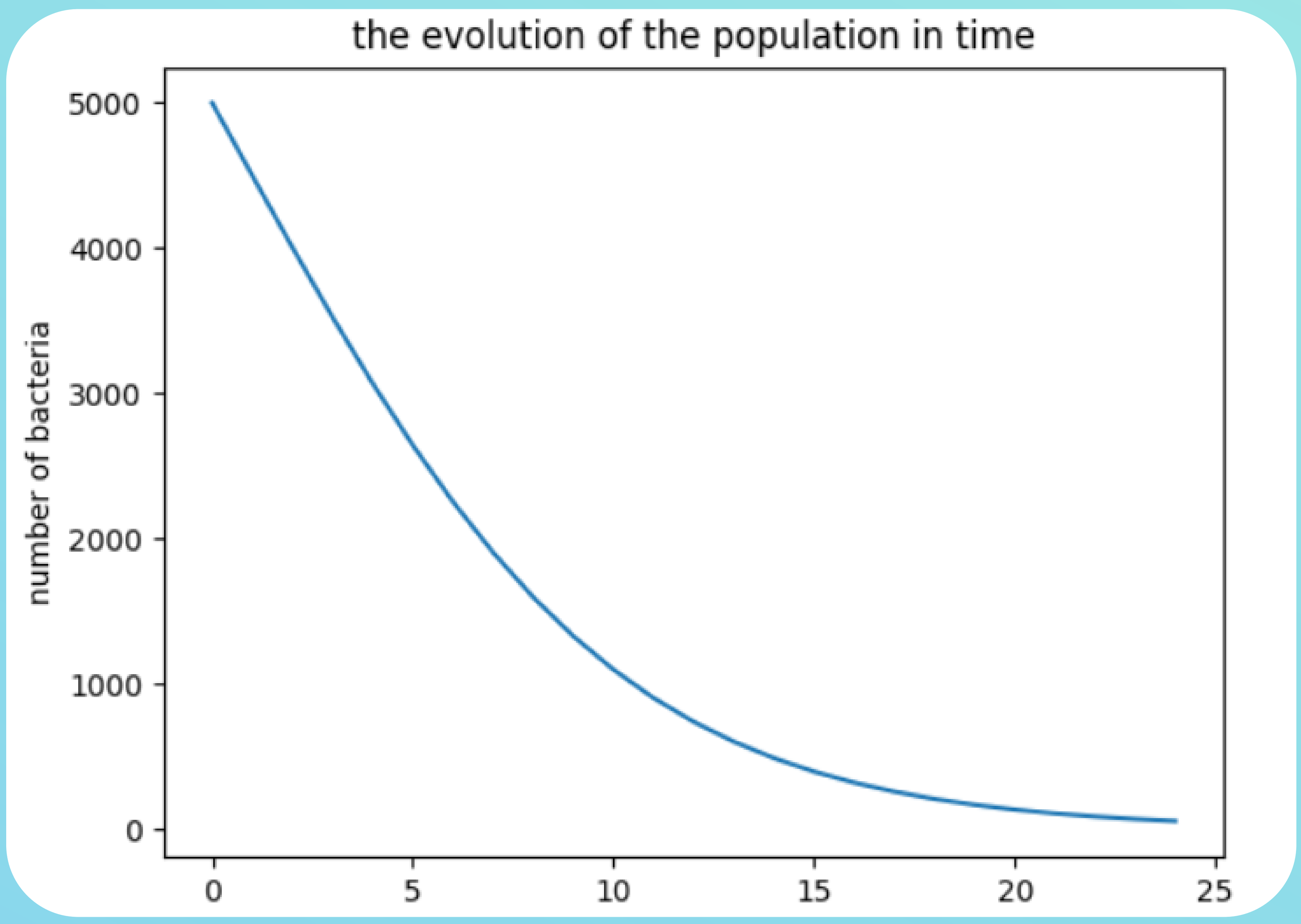
Doing multiple tests, we came to the conclusion that the population always ends up dying. An explanation for this is that the toxins always accumulate no matter what, so it will always grow. However, the bacteria can also die if there are too many toxins, so there will always be a moment when they stop growing and the population declines.

Mathematically, T(t) is always growing => 1-aT(t) will reach a point where it is negative => dN wil become negative => N will always start to decrease at some point. Eventually it will reach 0.

Conclusion

By not letting the toxins decay or evaporate in some way, the bacteria population will always die. If the decay is taken into account, the number of bacteria and the toxin quantity will reach a stable equilibrium.

For future ideas, it would be interesting to change the equations and see what the outcomes are.

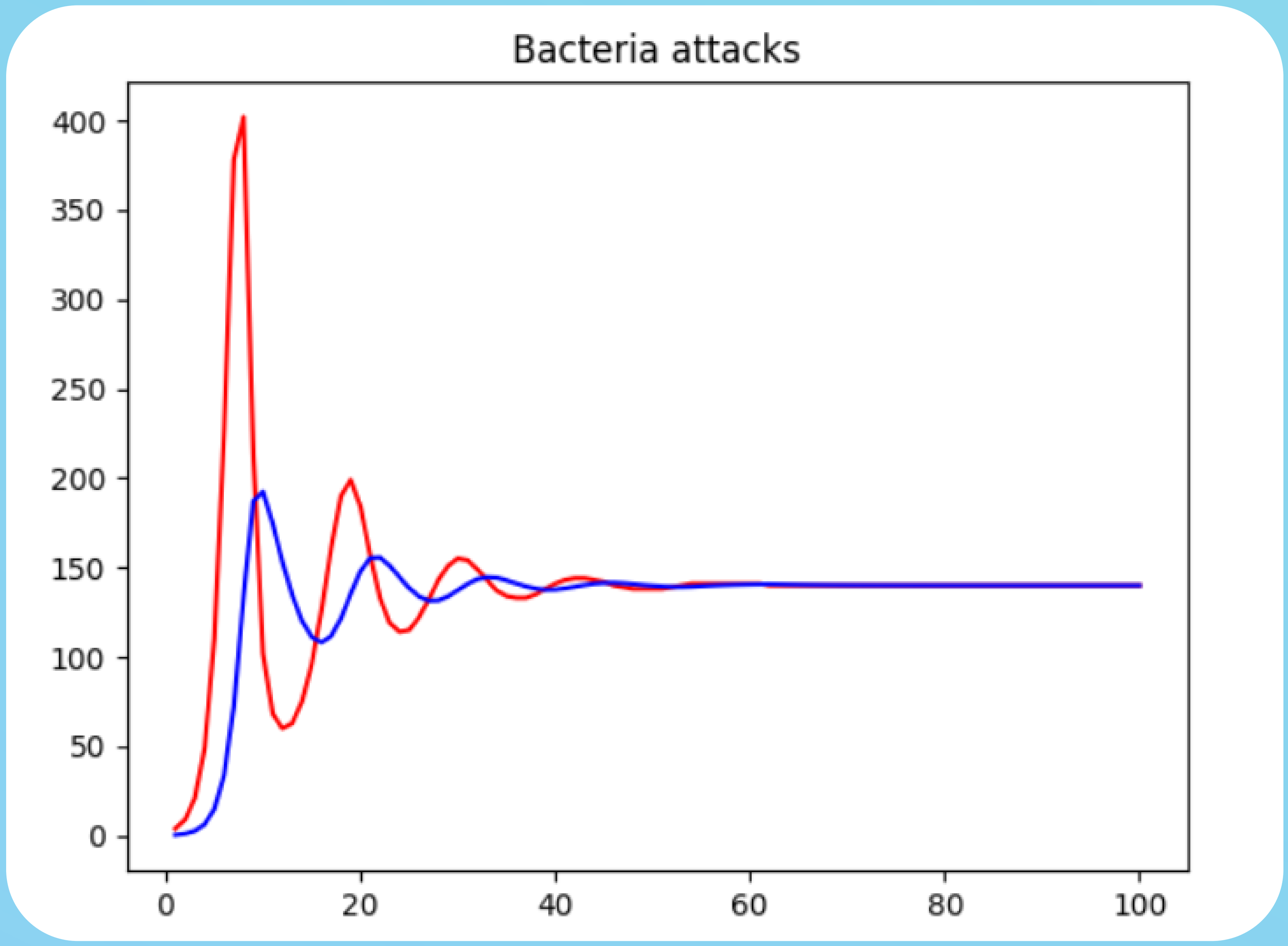


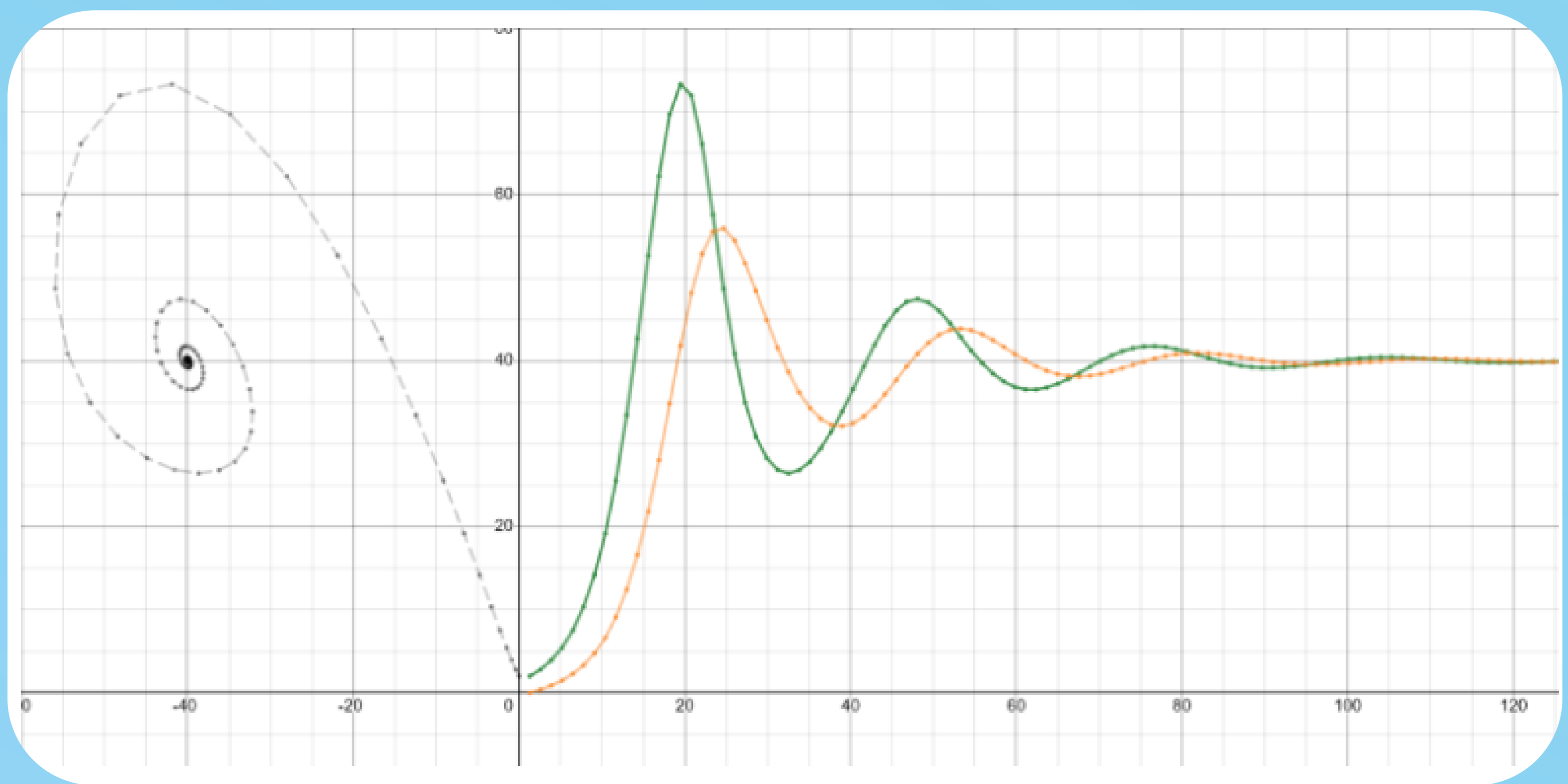
Alternate problem

Changing the initial problem by introducing a rate of toxin decay, makes the solution more interesting. The two new equations we used were

$$dN = k \cdot N(t) \cdot (1 - a \cdot T(t))$$
$$dT = r \cdot N(t) - b \cdot T(t)$$

In this way, the toxins decay proportionally to the current quantity. With these equations, the simulations were a bit different:





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