

The Kaprekar process

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1. The problem:

- We take an **integer (with 2 digits for the explanations)**;
- We put their digits in a **descending order**: it give us $N1 = \mathbf{ba}$ (such as $\mathbf{b} \geq \mathbf{a}$);
- Then we put the same digits in an **ascending order**: we get $N2 = \mathbf{ab}$ (such as $\mathbf{a} \leq \mathbf{b}$);
- After we **subtract** $N1$ by $N2$, it give us the difference D such as $\mathbf{D} = \mathbf{N1} - \mathbf{N2} = \mathbf{ba} - \mathbf{ab}$.

And and we do the algorithm again with the difference D until we find a loop or an end, we can complete the number with 0 at the left in order to always start with an integer with the same number of digits (2 in this example).

1. Results and speculations:

For three and four-digit numbers, we found a constant (a closed loop):

- 495 for the three-digit numbers;
- 6174 for the four-digit numbers.

For every numbers that is composed of the same digits (also called rep-digit), the result we get is 0.

And for the two-digit numbers, we found a loop but not a closed one.

1. Research:

3.1 With two-digit numbers:

For the **two-digit numbers** we did it manually until 99.

We can see it works for every numbers except **rep-digit** like 11, 22, 33, 44, 55 ... 99. For this numbers the result is 0. So the proceed works only if $\mathbf{N1} \neq \mathbf{N2}$ and if $\mathbf{a} \neq \mathbf{b}$.

At some point, the result of the loop is always **9** and that every result is a **multiple of 9**. So the results before finding 9 are 18, 27, 36, 45, 54, 63,72,81 and 90.

The loop seems to never be closed because even though we end with number 9, a new loop starts all

over again. We chose the calculation $90 - 09 = 81$ to be the “end” of the loop because if we continue this expression will come again.

Up to 99, we notice that the calculation $90 - 09 = 81$ comes up after **two calculations** nine times in a row, every time the digits are following each other respecting the **increasing order** (0123456789). We will get 9 after completed the process **three times**: 16/61, 17/71, 27/72, 28/82, 38/83, 39/93, 49/94, 50/05 and 60/06; after **four times**: 14/41, 19/91, 25/52, 30/03, 36/63, 47/74, 58/85, 69/96 and 80/08; for 15/51, 18/81, 26/62, 29/92, 37/73, 40/04, 48/84, 59/95 and 70/07 we found 9 after **five times** and after **six calculations** with 13/31, 20/02, 24/42, 35/53, 46/64, 57/75, 68/86 and 79/97.

Another option to consider was to take a two-digit number (N1) and subtract **b** with **a**. You multiply the result by 9 and it gives you the difference.

$$\begin{aligned} \text{Ex: } 48 &= \mathbf{ab} & \mathbf{b} - \mathbf{a} &= 8 - 4 = 4 \\ & & 4 \times 9 &= 36 \\ & & 84 - 48 &= 36 \end{aligned}$$

3.2 With three-digit numbers:

For the three-digit numbers we used a program on excel and checked for numbers up until 999.

Then we found an expression:

$N1 = \mathbf{cba}$ and $N2 = \mathbf{abc}$ with $\mathbf{c} \geq \mathbf{b} \geq \mathbf{a}$:

$$\begin{aligned} N1 - N2 &= (\mathbf{c} \times 100 + \mathbf{b} \times 10 + \mathbf{a} \times 1) - (\mathbf{a} \times 100 + \mathbf{b} \times 10 + \mathbf{c} \times 1) \\ &= \mathbf{c} \times 100 + \mathbf{b} \times 10 + \mathbf{a} \times 1 - \mathbf{a} \times 100 - \mathbf{b} \times 10 - \mathbf{c} \times 1 \\ &= 100\mathbf{c} + 10\mathbf{b} + 1\mathbf{a} - 100\mathbf{a} - 10\mathbf{b} - 1\mathbf{c} \\ &= 99\mathbf{c} - 99\mathbf{a} \\ &= 99(\mathbf{c} - \mathbf{a}) \end{aligned}$$

This expression works if and only if $\mathbf{c} \neq \mathbf{a}$ and if \mathbf{c} , \mathbf{b} and \mathbf{a} are **positive numbers**.

For every calculation with three-digit numbers, there is an **end** (unless $\mathbf{c} = \mathbf{a}$).

$954 - 459 = 495$ is the end of the loop because it's the same digits that come back $954 - 459 = 495$. So **the loop is closed**. When we end up with a **constant or a dead end**, we say the loop is closed. Therefore, we found a constant for three-digit numbers: **495**. That is true, except for **rep-digit**. If we found **99**, we can just complete with a **0** to restart the program.

3.3 With four-digit numbers:

For four-digit numbers we used a program on excel too, and checked for numbers up until 9999. Then we tried again an expression based on the same “model” than the others, but it became too difficult to use. This is what we got:

$$\begin{aligned} &(\mathbf{d} \times 1000 + \mathbf{c} \times 100 + \mathbf{b} \times 10 + \mathbf{a} \times 1) - (\mathbf{a} \times 1000 + \mathbf{b} \times 100 + \mathbf{c} \times 10 + \mathbf{d} \times 1) \\ &= \mathbf{d} \times 1000 + \mathbf{c} \times 100 + \mathbf{b} \times 10 + \mathbf{a} \times 1 - \mathbf{a} \times 1000 - \mathbf{b} \times 100 - \mathbf{c} \times 10 - \mathbf{d} \times 1 \\ &= 1000\mathbf{d} + 100\mathbf{c} + 10\mathbf{b} + 1\mathbf{a} - 1000\mathbf{a} - 100\mathbf{b} - 10\mathbf{c} - 1\mathbf{d} \\ &= 999\mathbf{d} + 90\mathbf{c} - 999\mathbf{a} - 90\mathbf{b} \\ &= 999(\mathbf{d} - \mathbf{a}) + 99(\mathbf{c} - \mathbf{b}) \end{aligned}$$

For every calculation with four-digit numbers, we found $7641 - 1467 = 6174$. **6174** is the **constant** for four-digit numbers, it's the end of the loop, because it's the same numbers come back. So **the loop is closed**. Except for the **rep-digit**. If we found **999**, we can just complete with a **0** to restart the program.

2. Conclusion

In conclusion, we saw that for **three and four-digit numbers** there is a **constant** (a closed loop), for every number that is composed of the same digits (also called **rep-digit**), the result we get is **0**, and for the **two-digit numbers**, we found a **loop but not a closed one**. There is no difference if the number is odd or even and there is no difference either if it's a prime number.